Low Lunar Orbit Design via Graphical Manipulation of Eccentricity Vector Evolution

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Low lunar orbits, such as those used by GRAIL and LRO, experience predictable variations in the evolution of their eccentricity vectors. These variations are nearly invariant with respect to the initial eccentricity and argument of periapse and change only in the details with respect to the initial semi-major axis. These properties suggest that manipulating the eccentricity vector evolution directly can give insight into orbit maintenance designs and can reduce the number of propagations required. A trio of techniques for determining the desired maneuvers is presented in the context of the GRAIL extended mission.

Nomenclature

e	=	eccentricity
ω	=	argument of periapse
ΔV	=	propulsive change in velocity

I. Introduction

LOW lunar orbits, such as those used by the Gravity Recovery and Interior Laboratory (GRAIL) mission, experience significant variations in their eccentricity and argument of periapse due to the non-sphericity of the lunar gravity field. Of all the classical orbital elements, only these two elements undergo significant secular changes; the semi-major axis and inclination oscillate around their mean values over periods of weeks, as does the longitude of the ascending node for a nearly polar orbit such as GRAIL's. This means that the evolution of a low lunar orbit can be represented in a single two-dimensional plot which shows the path of the eccentricity vector over time, where the eccentricity vector is a vector which points from the center toward the periapse and whose length is equal to the eccentricity (so that the values of eccentricity and argument of periapse form a polar coordinate system for eccentricity vector space, which we call e-ω space). The eccentricity vector defines where the orbit is positioned and oriented within the orbit plane.

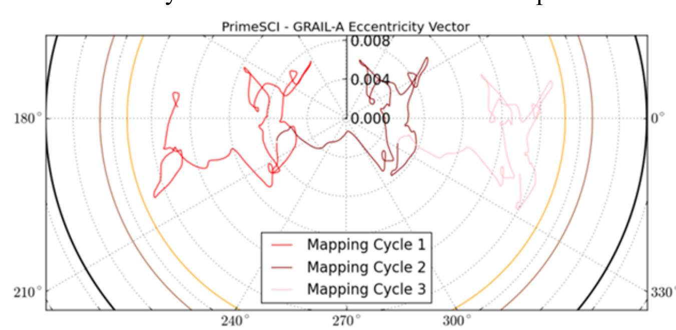

Figure 1. The GRAIL primary science mission orbit experiences three repetitions of its e-ω evolution.

The key to the design of the GRAIL extended mission is that this variation, or evolution, follows a pattern that nearly repeats every 27.3 days, the length of a lunar sidereal month. This repetition in the evolution of the eccentricity vector is clearly seen in Figure 1 (and would not be apparent if eccentricity and argument of periapse were taken to be Cartesian coordinates). The initial eccentricity and argument of periapse correspond to the point on the upper left and the general evolution of the eccentricity vector is from left to right in e-ω space. This evolution will, without intervention, eventually drive periapse below the surface. The lower the mean orbit altitude, the faster this occurs. The GRAIL prime science orbit altitude was 55 km, and the initial eccentricity (e) and argument of periapse (ω)

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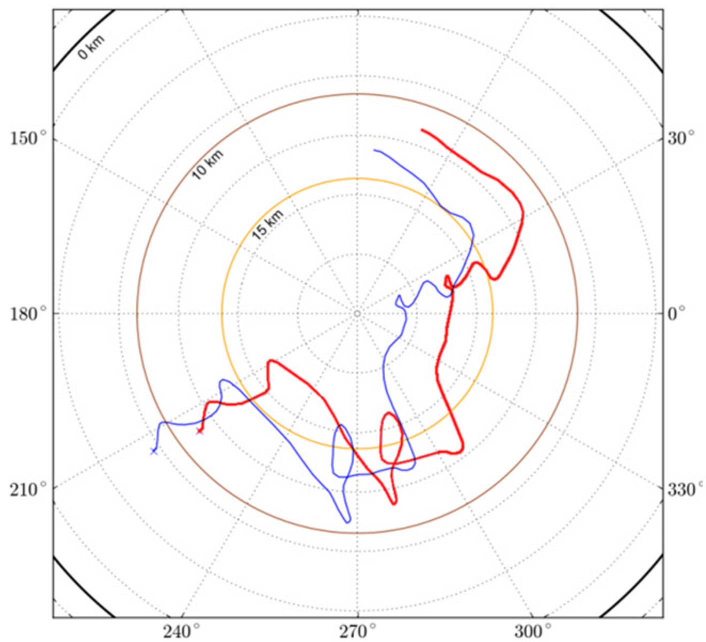

Figure 2. Changes in the initial *e-ω* point (marked with an x) do not change the major structure of the *e-ω* evolution.

were chosen such as to maximize the lifetime of the orbit without maneuvers. This resulted in a 100-day maneuver-free lifetime.

The repetition implies that the evolution of the orbit depends only on time, which in this case is a proxy for the longitude of the ascending node of the orbit in Moon-fixed coordinates. The evolution of the position of the orbit in the orbit plane depends on its orientation with respect to the gravity field of the Moon. Somewhat surprisingly, the evolution is nearly invariant with respect to its initial values of eccentricity and argument of periapse, where these values are taken at the periapse of each orbit so that the short-term variations within each orbit are ignored. This invariance is demonstrated explicitly in Figure 2, which plots the evolution of these values. The key observation is that the vector difference in the *e-ω* point between the two evolutions at any point in time is nearly the same as the difference in the initial *e-ω* points. Stated another way, the *e-ω* state transition matrix is nearly constant. By taking advantage of this near-invariability, it becomes possible to design a low orbit that requires periodic maintenance to remain above the surface by graphically manipulating the evolution and thus avoid unnecessary propagations.

Three techniques were explored and used in the development of the GRAIL extended mission.[1] The primary objective was to design an orbit as low as possible with as small an altitude variation as possible (i.e. minimize the value of the maximum eccentricity) without incurring large operations costs or risks. The largest driver of operations costs is maneuver frequency. All three techniques take advantage of the near-invariability of the *e-ω* evolution by assuming it is entirely invariable at some stage in the process. The first technique is a purely graphical technique for estimating the relationship between maneuver frequency and maximum eccentricity variation, as well as a first order estimate of the required ΔV. The second and third techniques are hybrid graphical/numerical in nature and take advantage of the invariability to significantly reduce the number of propagations required. The second technique determines the set of *e-ω* targets for the maneuvers to minimize the altitude variation between maneuvers. This results in a lower-bound on the maximum eccentricity. By setting the maximum eccentricity to some specific altitude, the mean altitude across a sequence of maneuvers can be determined. The third and final technique is the most computationally intensive and minimizes the ΔV of the maneuver set given a constraining circle in *e-ω* space.

II. Estimating Maneuver Frequency vs. Maximum Eccentricity

The first technique was developed to explore the relationship between maneuver frequency and maximum eccentricity. The technique was to take a single 27.3 day cycle of the *e-ω* evolution of the primary mission and lay a series of minimally-overlapping equal-sized circles down on it until the entire evolution was covered. This process can be done using any number of graphical tools, from drafting tools to high-end graphics software such as Adobe Photoshop. The result defines a trajectory option and determines the maneuvers needed to implement it: each time the trajectory enters a new circle, a maneuver changes the eccentricity and argument of periapse so that the pattern of the trajectory within the circle would move over to be centered at the origin (where the eccentricity is zero). The smaller the circle, the smaller the maximum value of the eccentricity along the trajectory within the circle after it is centered and thus the smaller the altitude variation of the resulting orbit design. On the other hand, the smaller the circle, the more circles would be required, and the more circles, the more maneuvers.

There are several ways of laying down these circles. First, the designer may select a fixed circle diameter and determine how many are required to cover a 27.3 day cycle. Alternately, the designer may select a number of circles (maneuvers) desired for a 27.3 day cycle and attempt to determine the smallest circle size required to cover the cycle, while allowing the circles to vary in size. Finally, the second approach may be used, but with equal circle sizes. It was this final method that was used to find how circle size (and thus altitude variation) depends on the number of maneuvers. The last two methods are somewhat more difficult, as they have more degrees of freedom within this purely manual technique.

Figure 3 illustrates how this process works for various numbers of circles, using magnified views of the central section of Figure 1, with dots marking the first periapse of each day. On the left, we started with a single circle and found the location that would cover 27.3 days with the smallest circle; implementing this would involve a reset maneuver when the orbit plane is in the same Moon-fixed position as on the fourth day of a primary-mission mapping cycle; the reset maneuver would center the circle and would be followed by monthly reset maneuvers thereafter. The actual altitudes attained with this design in an orbit with an average altitude of 28 km are shown in the altitude plot below the orbit evolution plot. This design would allow a reduction of 27 km from the primary mission average altitude of 55 km and would mean that the extended mission could have half the altitude and altitude variation of the primary mission.

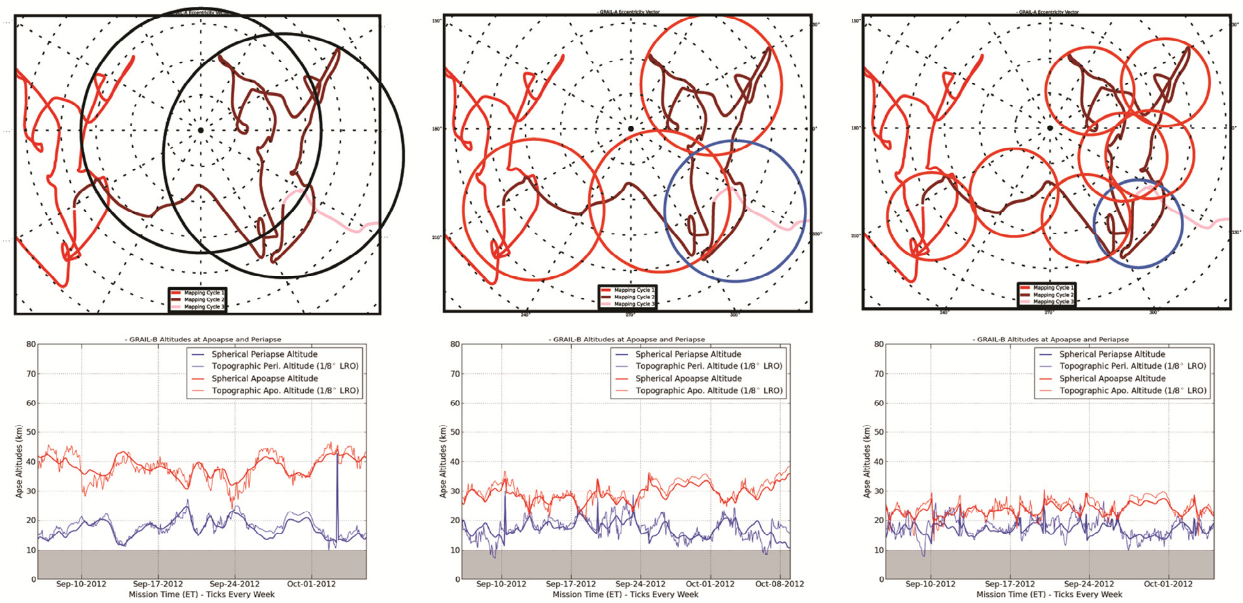

Figure 3. Circles in e-ω space illustrate 1, 3, and 7-maneuver-per-month solution and the resulting altitudes (thicker lines are relative to a spherical moon, while the thinner lines are relative to a topographic map).

The basic shape of the orbit evolution means that not much reduction in circle size (hence altitude variation) can be achieved by using two circles to cover the pattern. Three circles, however, do yield a significant further reduction as shown in the center plots of Figure 3; the average altitude can be further reduced by 5 km. The blue fourth circle in the evolution plot covers the same subpattern as the first circle and is itself the first circle of the next mapping cycle. After the three-circle solution, the shape of the evolution does not permit much reduction in circle size until the number of circles reaches seven, in which time the average altitude could drop another 3 km. This solution is shown in the right-hand plots of Figure 3, where again the blue circle is the first circle of the next mapping cycle. Note that in both multi-circle solutions, the reset maneuvers are irregularly spaced; in the seven-circle solution in particular, the second circle is traversed in less than two days, while the fifth circle covers more than six days of orbit evolution.

As the objective is to keep the orbit average altitude as low as feasible, the reset maneuvers need to change the eccentricity vector without changing the size of the orbit. There are two ways of doing this when the orbit evolution moves into the overlap between successive circles. The first is similar to Hohmann transfers between coplanar circular orbits—a tangential maneuver at apoapse approximately circularizes the orbit and then an opposite tangential maneuver at a true anomaly approximately 180 degrees greater than the desired argument of periapse restores the initial semi-major axis and sets the eccentricity vector as desired. The intermediate orbit can be exactly circular if the initial and final values of eccentricity are the same. This strategy is illustrated in Figure 4. Alternatively, radial maneuvers can be applied where the initial and final orbits intersect to move the orbit from the end of one segment (which is what we call the *e*-*ω* evolution within a circle) to the beginning of the next segment. By measuring the distance between the centers of the various circles, the ΔV required for the maneuvers can be estimated. It has been shown that this distance is directly proportional to the ΔV if the maneuvers are purely radial.

It has been further shown that the total ΔV for a pair of tangential maneuvers is approximately half the ΔV for a single radial maneuver when moving an orbit in *e-ω* space.[2]

Using this technique, then, it becomes possible with a single propagation to determine the relationship between maneuver frequency and maximum eccentricity and to estimate the ΔV needed to implement the trajectory. Through the maximum eccentricity, the minimum achievable altitude achievable can be determined as described above. The analysis shown in Figure 3 led to a conclusion that maneuvering the spacecraft more than once a month was justified, but that there was a point of diminishing returns. Increasing the tempo much beyond three maneuvers/month did not lower the altitude enough. A weekly maneuver strategy was ultimately selected to make every segment exactly seven days long so that a constant weekly operational schedule could be followed.

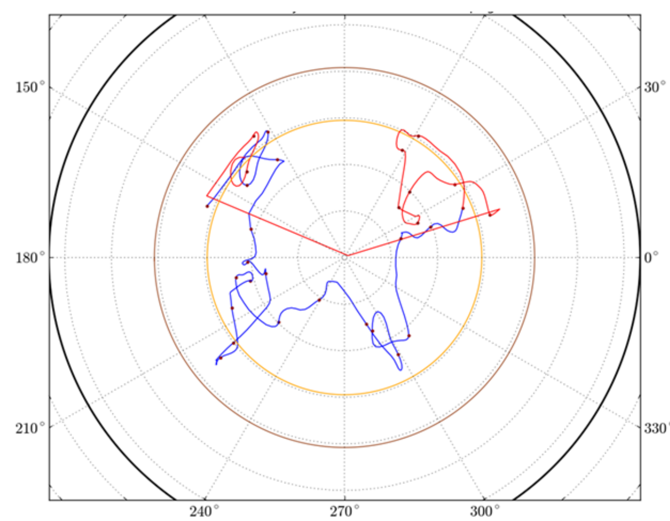

Figure 4. A Hohmann-like transfer to shift the *e-ω* evolution can reduce the required ΔV in half relative to a single radial maneuver. The last ten days of this orbit evolution are in red.

III. Minimizing Mean Altitude and Altitude Variation

The second technique was developed to determine the minimum circle size and minimum initial semi-major axis for a given maneuver frequency. The method began by choosing an initial *e-ω* point and semi-major axis, propagating for the designated duration, and then determining the minimum circumscribing eccentricity circle. The center of the circumscribing circle was determined numerically by minimizing the maximum distance from the center of a circle to each *e-ω* point within the segment. The centroid of the segment was used as an initial guess. The initial *e-ω* point was then shifted to center the circumscribing eccentricity circle in *e-ω* space. This process used only the values of the eccentricity and argument of periapse at periapse and did not require any additional propagation.

As the *e-ω* evolution is not truly invariant with respect to the initial *e-ω* point, the resulting set of *e-ω* targets will not necessarily result in the minimum altitude variation. But, since the variation between *e-ω* evolutions is small with small initial variation, the evolution was then re-propagated using the new *e-ω* point and a new shift in initial *e-ω* was determined. This was repeated until the change in the initial *e-ω* point was small. An example of the convergence history is shown in Table 1.

Table 1: Example Convergence History for Maximum Eccentricity Minimization

Iteration	Eccentricity	Argument of Periapse	Shift
1	7.917e-03	135.0	
2	2.798e-03	277.6	1.028e-02
3	2.782e-03	284.3	3.269e-04
4	2.709e-03	284.7	7.523e-05

This process was continued for each segment until the entire design was completed, as illustrated in Figure 5. Note in this picture that the first segment of each 28 day cycle (the first image in each column) is similarly shaped, but not identical. This is a consequence of the difference between 28 and 27.3 days. If the four segments in each month were instead 6.825 days long, they would be nearly identical. If desired, this entire process could be repeated with a different initial semi-major axis to ensure that the periapse altitude at maximum eccentricity was some desired minimum value. The resulting design minimized the maximum eccentricity over each segment, determined a floor value for the semi-major axis that would apply to all of the segments over the entire trajectory, and required 148 m/s of ΔV to implement, assuming radial maneuvers. The maximum eccentricity across the design and the ΔV budget is dependent on what day is chosen to do the maneuvers. Mondays (shown) break the segments into two

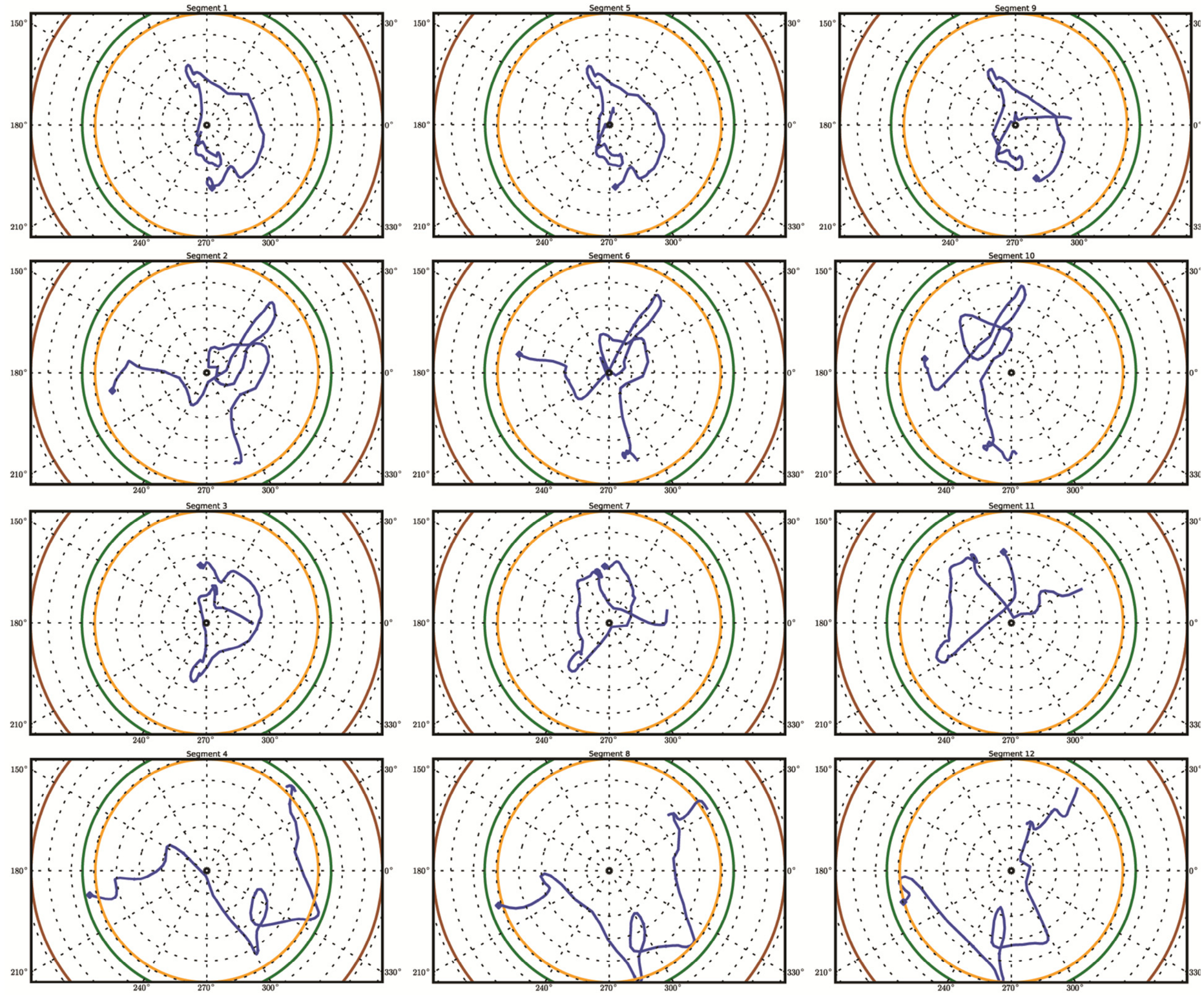

Figure 5. The 12 segments of this design, ordered by columns and each 7 days long, are positioned in *e*-*ω* space such that they are contained within the smallest possible value of *e*. The green circle is the largest value of e required by any segment, and the brown and yellow circles correspond to 10 km and 15 km periapse altitudes, respectively. Note the differences between the columns as a result of a cycle mismatch between 28 days and 27.3 days (the latter being the time for one lunar rotation underneath the orbit).

classes: the odd rows are more compact, and the even rows are less so. Thursdays, half a week later, have a more even distribution from one segment to the next, and result in a smaller maximum eccentricity as well as a smaller required budget as the total motion in *e*-*ω* space is reduced. However, sometimes operational considerations trump a more-optimal design and Monday maneuvers were chosen for the GRAIL extended mission. That way, a weekly schedule could be designed that would reduce operational risk by giving the weekend as margin in the maneuver design, test, and uplink process.

IV. Minimizing ΔV

The third technique was developed to minimize the ΔV required for the orbit design given a maximum circumscribing eccentricity circle. A design resulting from the technique described in Section III does not consider the fuel required to perform its maneuvers. If it is determined that the maximum eccentricity during the entire mission is what drives the desirability of an design, not the maximum eccentricity during each segment, then an additional degree of freedom becomes available to the designer. For example, in Figure 5, Segment 4 has the largest circumscribing eccentricity circle (the green circle). All of the other segments are within this green circle, but some (the odd segments in particular) are significantly more compact. By shifting the *e*-*ω* targets for these maneuvers, the total ΔV required can be reduced. This optimization can be achieved in a timely manner by taking advantage of the near-invariability of the e-ω evolution.

A constrained direct optimization problem was formulated to minimize the change in e-ω space across the discontinuities while constraining each segment to be within the maximum diameter circumscribing eccentricity circle. The controls in this problem were offsets to the initial e-ω points for each segment which had been determined through use of the second technique (Section III above). The cost function was the length of the discontinuities between the end of one segment and the beginning of the next multiplied by the mean semi-major axis of the orbit. This multiplication was performed to improve the numerical performance of the problem. The constraints were that no point in the e-ω evolution be outside a specified circle, which may or may not be centered in e-ω space. By assuming that each point in a segment would be offset in e-ω space by the same amount as the initial value, the controls could be optimized without additional propagations.

The constraints were simple to calculate, requiring that each value along the segment be added by the shift, but they present a difficult optimization problem. The value of the constraint is not necessarily continuously differentiable if more than one point along the segment is near the constraint bound, as in Segments 4, 8, and 12 (the bottom row) in Figure 6. This difficulty is not a feature of the invariability assumption, but is instead a consequence of applying a path constraint.

The optimization was performed using the *fmin_cobyla* function within the optimization package of the Python module *SciPy*[3] using the e-ω targets for the centered segments developed using the second technique as an initial guess. The *fmin_cobyla* function was selected because it was readily available, uses linear approximations of the cost and constraint functions (which are themselves nearly linear), and worked reasonably well. It typically required a large number of iterations to find a solution, but since no additional propagations were required during the optimization, this was not a significant penalty.

After the optimization problem converged, each segment would be re-propagated using the new initial e-ω point and the problem re-converged until the improvement in the cost function was small. This major iteration step achieved most of its improvement within in three or four iterations but did not converge to the 10^{-2} level until 8 iterations were completed, as shown in Table 2. It is believed that the back-and-forth during these tail-end iterations was caused by the change in the end-point of each segment integration, which changed due to the non-invariability of the evolution within the segment being of the same order as the changes to the initial point being applied by the optimizer.

If the optimization of the e-ω targets for each maneuver were to be designed using full propagations, 9,000 iterations would require 17,430 hours (24 months) of propagation time. Applying parallel computing to this problem would be of limited use, potentially reducing it to 1.7 months. Using this technique, an optimized ΔV was determined in only 18 hours without any parallelization. This represents a factor of 1000 improvement in the run time and reduced the 148 m/s from the "centered-segments" design by a third, to 100 m/s.

Table 2: Convergence History for DV Minimization

Major Iteration	Optimizer Iterations	Cost	Change in Cost	Change in Cost (%)
0	N/A	135.6		
1	8779	97.1	38.5	28.4%
2	3905	93.2	3.9	4.0%
3	4739	91.2	2.0	2.1%
4	7612	92.7	-1.5	-1.6%
5	5735	91.7	1.0	1.0%
6	4862	89.6	2.2	2.4%
7	4244	93.0	-3.5	-3.9%
8	7304	91.1	2.0	2.1%
9	5302	91.3	-0.2	-0.3%

V. Conclusion

The techniques described here were not the only ones used to design the low lunar orbits of the GRAIL extended mission, as they did not address the total suite of constraints applied to that design, such as ground-track spacing, topographic altitude constraints, and missed-maneuver considerations. However, they did provide the starting points for manual adjustments. The maneuver frequency studies resulting from Section II greatly pared down the trade space during initial orbit studies. The mean altitude from Section III was slightly adjusted to achieve the desired ground-track spacing and comparison of results for different segment-start days allowed the project to decide which one (Monday) to use. Finally, the results from the technique described in Section IV were manually adjusted in a few cases to satisfy the topographic and missed-maneuver requirements. These manual adjustments added 8 m/s to the overall design, which left more than sufficient margin for the GRAIL Extended Mission.

Acknowledgments

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